CCA Math Bonanza

March 8, 2025

Division II Team Round

T1) Byan Rai is deciding what to wear to school today. He has three shirts that are blue, red, and yellow, and he has four pairs of pants that are blue, yellow, green, and orange. He will pick one shirt and one pair of pants to wear, but he doesn't want both of them to be the same color. Find the number of ways for him to dress.

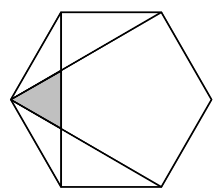
T2) The value

$$\frac{25!^2 - 24!^2}{25!^2 + 24!^2}$$

can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $100m + n$.

T3) Find the number of primes less than 100 which can be written in the form $a^4 + b^4$, where a and b are (not necessarily distinct) positive integers.

T4) The regular hexagon shown in the figure below has side length 1. The area of the shaded region can be expressed as $\sqrt{\frac{m}{n}}$, where m and n are relatively prime positive integers. Find $100m + n$.



T5) Alex picks a positive integer a, and Bryan picks a positive integer b. Alex's number has 2 digits both when written in base 12 and when written in base b. Find the sum of all possible values of b.

T6) Let $\triangle ABC$ be a triangle with $AB = 13$, $BC = 14$, and $AC = 15$. Let D be the foot of the altitude from A to BC. Let P be the point on segment AC such that $AP = AD$. Let Q be the point on segment AD such that PQ and BC are parallel. Find the area of $\triangle AQB$.

T7) Six boys and thirteen girls sit at a circular table. Over all possible ways for them to be seated, find the minimum possible number of people that have a girl to the left and right of them.

T8) Freddy the frog is currently standing at point $(1, 1)$ on the coordinate plane. Every minute, Freddy picks a cardinal direction (north, south, east, or west) uniformly at random and hops either 1 or 2 units, picking his hop length uniformly at random as well. He stops after hopping over either $x = 0.5, x = 2.5, y = 0.5$, or $y = 2.5$. The probability he hops over $x = 0.5$ or $y = 0.5$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $100m + n$.

T9) Find the number of positive integers n such that $n \leq 750$ and $\gcd(1500, n)$ is a prime number.

T10) Let d be a real number, and let a, b, and c be the roots of the polynomial $x^3 - dx^2 + 8x + 2$. Given that

$$\frac{ab}{a+b} + \frac{ac}{a+c} + \frac{bc}{b+c} = 2,$$

the value of d can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $100m + n$.