CCA Math Bonanza

March 8, 2025

Division I Team Round

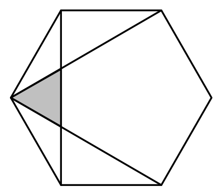
T1) Larry is making a burger. He starts with a bun and can add any combination of lettuce, cheese, tomato, onion, and meat, including the option to leave the burger plain with just the bun. However, he doesn't want to make a burger that has meat but no cheese. The order in which he adds his ingredients doesn't matter. Find the number of different burgers Larry can create.

T2) The value

$$\frac{25!^2 - 24!^2}{25!^2 + 24!^2}$$

can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $100m + n$.

T3) The regular hexagon shown in the figure below has side length 1. The area of the shaded region can be expressed as $\sqrt{\frac{m}{n}}$, where m and n are relatively prime positive integers. Find $100m + n$.



T4) The maximum possible value of $\frac{t}{3^t} - \left(\frac{t}{3^{t-1}}\right)^2$ over all real numbers t can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $100m + n$.

T5) Find the number of positive integers n such that $n \leq 750$ and $\gcd(1500, n)$ is a prime number.

T6) Three points A, B, and C are chosen uniformly at random along the circumference of a circle of radius 1. The probability that all angles in $\triangle ABC$ are less than $75°$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $100m + n$.

T7) Let a, b, and c be real numbers that satisfy the following system of equations:

$$(a+1)(b+1)(c+1) - abc = 4$$
$$(a-1)(b-1)(c-1) + ab + bc + ca = 4$$
$$a + b + c + \frac{ab}{c} + \frac{bc}{a} + \frac{ca}{b} = 0$$

The sum of all possible values of $a^2 + b^2 + c^2$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $100m + n$.

T8) Let $\triangle ABC$ be a triangle with $BC = 12$, $\angle BAC = 60°$, and $\angle ABC = 75°$. Let O, H, and I be the circumcenter, orthocenter, and incenter of $\triangle ABC$, respectively. If O' is the reflection of O over BC, then the area of $O'HIO$ can be expressed as $\sqrt{v} - \sqrt{w}$, where v and w are positive integers. Find $100v + w$.

T9) Call a function $f(x)$ which takes in rational numbers as inputs and outputs real numbers *awesome* if, for any rational number x, $f(x) = f(2x) + f\left(\frac{x}{2}\right)$, and for any odd integer k, $f(k) = k$. Let S be a subset of $\{1, 2, ..., 100\}$ such that $\sum_{n \in S} f(n)$ has exactly one possible value across all *awesome* functions f. Find the maximum possible size of S.

T10) Let $\triangle ABC$ be a triangle with $BC = 120$ and $\angle BAC = 60°$. Let M be the midpoint of BC. It is given that $AM = 90$. Let Ω be the circumcircle of $\triangle ABC$, and let D be the midpoint of major arc BC. Let line AM intersect Ω at E, line DE intersect BC at F, and line AF intersect Ω at G. Let $K \neq G$ be the intersection of Ω with the circumcircle of MFG. Find AK.