CCA Math Bonanza

March 8, 2025

Division I Tiebreaker Round

TB1) Let n and x be positive integers such that the sum of the digits of x is 2 and

$$9900\,9900\,9900\,9901 \times n = x.$$

Find the smallest such n.

TB2) Charlotte the cat lives in Cartesia, a city on the coordinate plane whose roads are the lines $x = a$ and $y = a$ for integers a. Charlotte is currently standing at the origin, and would like to walk to the highway at $y = 5$, and then to her home at $(6, 1)$. Let D be the length of the shortest possible path she could take home. Find the distinct number of paths of length D Charlotte can take.

TB3) A positive integer n is *x-central* if there exists integers a and $-2 \leq b \leq 4$ such that $n = ax + b$. Find the number of positive integers $k < 8390$ that satisfy exactly 2 of the following:

- k is 13-central
- k is 17-central
- k is 19-central

TB4) Unit cube $ABCD - EFGH$ has square faces $ABCD$ and $EFGH$, with vertices A, B, C, D adjacent to vertices E, F, G, H, respectively. Two regular tetrahedrons with bases DBE and DBG are constructed. Let the apexes of the two tetrahedrons be P and Q, with both P and Q lying outside the cube. The length PQ can be expressed as $\sqrt{\frac{m}{n}}$, where m and n are relatively prime positive integers. Find $100m + n$.